

## CAS CS 548. Problem Set 1

**Due 5 pm Friday, February 17, 2006, in the drop box near the CS office.**

**Problem 1.** (Cramer-Shoup)

(a) The Cramer-Shoup encryption scheme requires “proof” that  $\log_g a = \log_{\hat{g}} \hat{a}$ . However, it is very straightforward for the decryptor to check that: just check that  $a^w = \hat{a}$ . This requires simply including  $w$  into the secret key. Suppose we modify the Cramer-Shoup scheme by adding  $w$  into the secret key and replacing step **D4** on p. 21 with checking if  $a^w = \hat{a}$ . (Thus, the values  $d, e, f$  are no longer used.) Show exactly where (by pointing out what game and what lemma and giving an explanation) would the Cramer-Shoup proof of security break down for such a modified scheme.

(b) Let us now focus on the question of why any consistency check at all is required. Consider now a different modification to the Cramer-Shoup scheme: simply omit the check of consistency altogether, i.e., omit steps **D3** and **D4**. Modify the security proof accordingly, omitting step **D4'** in games  $\mathbf{G}_3$ ,  $\mathbf{G}_4$  and omitting game  $\mathbf{G}_5$  entirely. Where does the proof break down now?

(c) Suppose  $G$  is a DDH group of size  $q$ . Show that the usual (two-generator) DDH assumption tightly implies hardness of three-generator DDH. More precisely, suppose that any adversary running in time  $t$  can't distinguish  $(f, g, f^u, g^u)$  from  $(f, g, f^u, g^v)$  (for random  $f, g \in G, u, v \in \mathbb{Z}_q$ ) with advantage greater than  $\epsilon$ . Show that then any adversary running in time  $t'$  can't distinguish  $(f, g, h, f^u, g^u, h^u)$  from  $(f, g, h, f^u, g^v, h^w)$  (for random  $f, g, h \in G, u, v, w \in \mathbb{Z}_q$ ) with advantage greater than  $\epsilon'$ . Make this a tight reduction:  $t'$  and  $t$ , as well as  $\epsilon'$  and  $\epsilon$ , should differ only by small additive amounts. Hint: this is buried in the Cramer-Shoup proof.

**Problem 2.** (CCA2 out of CPA) Consider the following attempt at constructing an IND-CCA2 public-key encryption scheme: start with an IND-CPA public-key encryption scheme (Gen, Enc, Dec) with keys  $(pk, sk)$  and a *deterministic* MAC (i.e., for each  $m, K$ , there is only one valid tag). To encrypt  $m$ , generate a random  $K$ , set  $c_1 = \text{Enc}_{pk}(m, K)$  and  $c_2 = \text{MAC}_K(c_1)$ . To decrypt, get  $m, K$  using  $\text{Dec}_{sk}$  and reject if the MAC doesn't verify. Demonstrate via a counterexample that the resulting scheme is not necessarily IND-CCA2 secure.