Due: Friday, September 24 by 5:00pm - submit via Gradescope

Reading: For matrices read pages 75-82 about Strassen’s algorithm, and also look through Appendix D.
For polynomials start reading Chapter 30 on the FFT in the textbook, pages 898 - 915.

Problems: Please limit your answer to the following problems to at most 1/2 a pages each.

1. i. You are given the point-value form of a polynomial consisting of the 3 points (1,-7), (-2, -7), and (-1, 7).

Use the interpolation formula of Lagrange (found on page 902, equation 30.5) to find a polynomial $A$ of degree 2 which goes through those 3 points.
Show your work.

Answer: The polynomial $A(x) = -7x^2 - 7x + 7$.

You should show some of the Lagrange interpolation formula on the 3 points given. Enough to get at least part of the answer.

ii. Is the polynomial $A$ you found in (i) the unique polynomial of degree 2 which goes through the 3 points? Why or why not?

Answer: Yes, the three given points of $A$ and not on any line and a degree 2 polynomial is uniquely characterized by 3 non-collinear points.

iii. Could you find degree a one polynomial which goes through these same 3 points? How about a degree four polynomial? Why or why not?

Answer: A degree 3 or degree 4 polynomial which goes through these same 3 points is $A(x)(x-t)$ or $A(x)(x-t)^2$.

2. i. Prove that for any positive even $n$, $\omega_n^{n/2} = \omega_2 = -1$.

ii. List all the principal $6^{th}$ roots of unity, and $7^{th}$ roots of unity.

iii. Show that if $p$ is prime then every $p^{th}$ root of unity other than 1 is principal.

3. i. Recall the usual algorithm we use to multiply two $4 \times 4$ matrices of integers.

Exactly how many regular integer multiplications does this take? How many integer additions?
ii. Now do the same problem as in problem i. but this time use Strassens algorithm and divide and conquer to do the 4×4 multiplication. Make sure you use Strassen's algorithm at all places of the divide and conquer tree where you do the multiplications.

Answer the same two questions as in part i.

Answer: i. If we do the 4 by 4 matrix multiplication we use $4^3 = 64$ multiplications and $4^23 = 48$ additions.

ii. Strassen, on the other hand, takes 7 multiplications and 18 +'s to multiply two 2×2 matrices. So to multiply two 4×4 matrices we use $7 \times 7 = 49$ mults and $40+126+32 = 198$ +’s. The work to justify all these numbers is not shown here. However, as a hint, the number of additions for Strassen’s algorithm is $198 = 40+126+32$ where $40 = 10 \times 4$, $126 = 7 \times 16$ and $32 = 8 \times 4$. 