Source-Matched Spreading Codes for Optical CDMA

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***Abstract*— Puncturing is studied as a physical layer mechanism for efficiently transmitting datastreams containing bits of unequal priority via wavelength-time optical CDMA. Puncturing increases system capacity, while ensuring that important bits are received with low BER.**

***Index Terms*— Optical communication, Code division multi-access.**

I. Introduction

IN A wavelength-hopping, time-spreading optical CDMA (OCDMA) system (e.g. [1], [2]) enables multiple access communication on a shared optical media by creating channels through spreading of each user's data in wavelength and time. In this letter we describe an interesting property of OCDMA networks; the ability to match the spreading code to source data so that the capacity of the optical network is increased. When an analog signal is source-encoded (*i.e.* quantized and compressed) for transmission via a digital system, practical source encoders cannot remove all redundancies in the signal, so that the resulting data stream will contain substreams of varying significance. As such, the impact of channel errors on the message depends strongly on what portion of the data stream was corrupted. (As a simple example, consider pulse-coded modulation of speech signals, where the importance of the most significance bit of each sample outweighs that of the least significant bit. Furthermore as [3] indicates, the effect of channel errors on more complex compression schemes such as subband coding and discrete cosine transform depends strongly on the energy compaction of the bits corrupted by the channel.) In these situations, a priority scheme may be defined to make effective use of limited system resources while ensuring that the more important substreams are encoded so that they are transmitted with higher reliability than the less important substreams. There is a considerable body of literature on the idea of matching a channel code to the priority levels in source data, for a range of applications including image, video, and speech coding (see for example [3] and references therein).

The most common way to transmit data with unequal levels of reliability is to subject the more important substreams to better error control codes than the less important substreams. Alternatively, a single block unequal error protection (UEP) code may be used (see [4] for the earliest UEP codes). With block UEP codes, data is organized into blocks of length $u_M + u_L$ with u_M more important bits and u_L less important bits (the scheme can similarly be extended to blocks containing more than two priority levels), and is treated with a single error control code. The portion of the codeword associated with the more important bits is separated by a greater distance, and therefore provides better error protection, than the code sequences associated with the less important bits. The implementation of error control codes in high-speed optical communication systems can be difficult due to the level of complexity imposed by encoding and decoding. We propose instead a scheme that works directly at the physical layer of a incoherent, asynchronous, wavelength-hopping time-spreading OCDMA network to enable transmission of unequal priority substreams with unequal reliability, so that the capacity of the network is increased. We begin by describing a scheme which uses puncturing of the carrier hopping prime codes in [5]. We then analyze the performance of our scheme and outline a possible implementation.

II. Punctured Carrier Hopping Prime Code

The carrier hopping prime code (CHPC), described in [5], is a set of $w \times p_1 p_2 ... p_k$ (0,1)-matrices or *codewords* (for $1 < w \leq p_1 \leq p_2 \leq ... \leq p_k$, all p_i prime numbers) where the w rows represent wavelengths and the $p_1 p_2 ... p_k$ columns represent timeslots. Each row of each matrix in the CHPC contains a single '1' element, representing a *chip* or optical pulse, so that for each codeword each chip is sent on a distinct wavelength.[1] We define the *correlation* between two w-row T-column (0,1)-matrices $\mathbf{x}$ and $\mathbf{y}$ as

$$C(\mathbf{x}, \mathbf{y}, \tau) = \sum_{j,k} \mathbf{x}(j,k)\mathbf{y}(j, k \oplus \tau)$$

where $\mathbf{x}(j,k)$, $\mathbf{y}(j,k)$ are entries in row j, column k of the matrices $\mathbf{x}$, $\mathbf{y}$ respectively, $\oplus$ denotes addition modulo T, and τ is an integer representing the relative cyclic shift

[1] The $w \times p_1 p_2 ... p_k$ CHPC consists of the codewords $\mathbf{x}_{i_1, i_2, ..., i_k}$ with $i_1 \epsilon \{0, 1, ... p_1 - 1\}$, $i_2 \epsilon \{0, 1, ... p_2 - 1\}$ ... $i_k \epsilon \{0, 1, ... p_k - 1\}$. If the ordered pair (j, t) represents the position of a '1' element in the CHPC codeword (where j specifies wavelength (row), and t represents timeslot (column)), each codeword $\mathbf{x}_{i_1, i_2, ..., i_k}$ may be specified by a set of w ordered pairs, where the j^{th} ordered pair is given by

$$(j,\, j \odot_{p_1} i_1 + (j \odot_{p_2} i_2) \cdot p_1 + ... + (j \odot_{p_k} i_k) \cdot p_1 p_2 ... p_{k-1}) \quad (1)$$

for $j = 0, 1, ... w - 1$. Note that "$\odot_p$" denotes modulo p multiplication. For more details, see [5].

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between matrices $\mathbf{x}$ and $\mathbf{y}$. Using this definition, the properties of the CHPC are zero auto-correlation sidelobes (*i.e.* for matrices $\mathbf{x}, \mathbf{y}$ in the $w \times T$ CHPC, $C(\mathbf{x}, \mathbf{x}, \tau) = 0$ for $\tau \neq 0$ (mod p)), an auto-correlation peak of height w (*i.e.* $C(\mathbf{x}, \mathbf{x}, 0) = w$) and maximum cross-correlation amplitude of unity (*i.e.* $C(\mathbf{x}, \mathbf{y}, \tau) \leq 1$ for all τ and $\mathbf{x} \neq \mathbf{y}$). The *cardinality* (the number of distinct codewords in the code) of the $w \times p_1 p_2 ... p_k$ CHPC is $p_1 p_2 ... p_k$.

Because the cardinality of the CHPC is related only to the number of columns (timeslots) in the CHPC, we can create CHPC of *Hamming weight* (the number of '1' elements in the matrix) $w \leq p_1$ by deleting a set of $p_1 - w$ rows (wavelengths) from each matrix in the $p_1 \times p_1 p_2 ... p_k$ CHPC. It follows that for $w + \Delta w \leq p_1, w \geq 1$, a weight $w + \Delta w$ CHPC constructed from $\{p_1, p_2, ..., p_k\}$, and a CHPC constructed from the same set of prime numbers and with weight w, are both valid CHPC, with correlation properties as described above and with cardinality $p_1 p_2 ... p_k$. This means that we can vary the weight of the CHPC on a *per user* basis, so that the BER of a particular user can be adjusted based on his or her priority in the OCDMA network (see our experimental verification of this in [6] and analysis in [7]).

The idea of puncturing on a *per bit* basis becomes useful when each user's data arrives at his or her transmitter in blocks consisting of some more important bits (*M-bits*) followed by some less important bits (*L-bits*) as in the UEP scheme in [4] . Consider a wavelength-hopping time-spreading OCDMA system that uses the $(w + \Delta w) \times T$ CHPC, where $w + \Delta w$ is the number of wavelengths, and T is number of timeslots used in the code. Assume that information is transmitted via on-off-keying, where the presence of a codeword indicates data bit '1' and the absence of a codeword indicates data bit '0'. In the sequel, we show that converting a weight $w + \Delta w$ CHPC to a weight w CHPC on a per bit basis by puncturing, or blocking, a set of Δw wavelengths when the L-bits are sent will cause the M- and L-bits to be received with unequal bit error rates (BER). The scheme can be easily extended to data blocks containing more than two priority levels. When our puncturing scheme is compared to transmission of the both M- and L-bits with an unpunctured CHPC of weight $w + \Delta w$, we show that the puncturing reduces multiple access interference and thus increases capacity.

III. Implementation and Performance Analysis

Following [5], we model an incoherent, asynchronous OCDMA multiple-access system that that uses on-off-keying of the CHPC and a correlation receiver to distinguish a particular user's code sequence from the arriving asynchronous wavelength-time OCDMA waveforms that share a passive optical media. When the correct user's data arrives at the receiver, an autocorrelation peak forms at the output of an optical correlator, which is the converted to an electrical signal by a photodiode and threshold-detected to recover the data bits. Note that [5] has shown that the optimal detection threshold for the CHPC is equal to its Hamming weight. (A possible realization of an incoherent, asynchronous OCDMA system that we have implemented in [2], [6] is shown in Fig. 1.) We

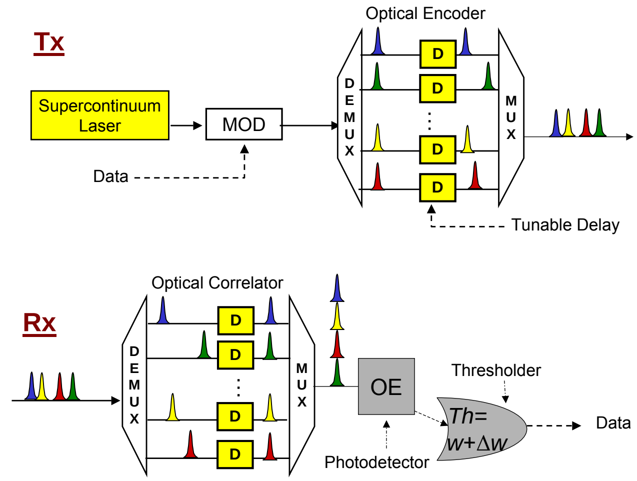

Fig. 1. An incoherent, asynchronous wavelength-hopping time-spreading OCDMA system due to [2]. *Transmitter*: A supercontinuum laser modulated via on-off keying is passed through an encoder. The encoder slices the supercontinuum into multiple wavelengths, aligns each pulsed wavelength to its appropriate timeslot using adjustable delay lines, and then recombines the wavelengths with a WDM multiplexer for transmission into the network. *Receiver*: The arriving signal passed through an optical correlator that is similar to the encoder, except that the wavelength and the time-delay arrangements are reversed. A photodiode converts the intensity of the signal at the output of the correlater to an electrical signal which is threshold-detected to recover the data bits.

assume here that system performance is limited by multiple access interference due to cross-correlation pulses created by interfering users at the output of the correlator, so that thermal and shot noise may be ignored. We further assume that all users are received with equal power so that, by the properties of the CHPC, the cross-correlation pulses have amplitudes of either zero or unity (as shown in [5]). For tractability we work with a discrete-time system where user transmissions are chip-synchronous, so that the chips (or timeslots) of different users are perfectly aligned although their bit frames (or codewords) may not be aligned. It is known (see [5]) that this assumption results in an upper bound of the continuous-time BER given by

$$P_{err} = \frac{1}{2} \sum_{\ell=Th}^{K-1} \binom{K-1}{\ell} q^{\ell} (1-q)^{K-\ell-1} \qquad (2)$$

where Th is the detection threshold, and $q = \frac{w}{2T}$ is the *hit probability* (for a $w \times T$ (unpunctured) CHPC), or the probability that the value of cross-correlation is unity for interfering user j at user i's receiver.

A. Some Puncturing Schemes

We can design each user's transmitter with Δw puncturing switches so that all users will puncture at the same Δw wavelengths on a per-bit basis. A more complex system, which we refer to as a "puncture with λ-shuffle" system, requires $w + \Delta w$ puncturing switches, so than all users may puncture any subset of Δw wavelengthes. (For the OCDMA encoder in Fig. 1, switches could be inserted immediately after the delay lines.) By allowing each user to puncture any of the

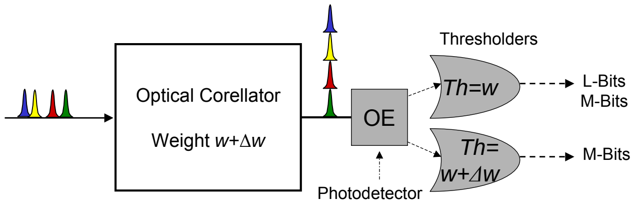

Fig. 2. The receiver for a punctured CHPC using one optical correlator and two electronic threshold detectors.

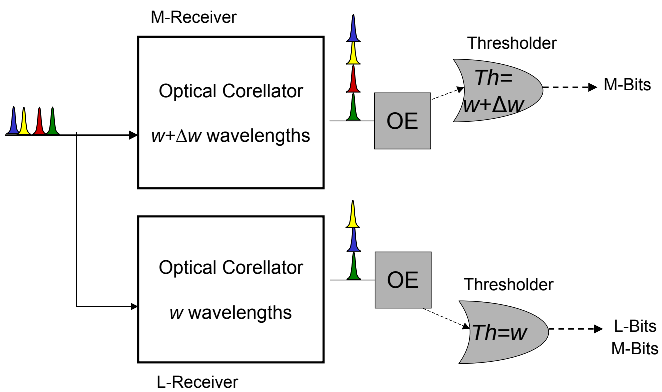

Fig. 3. The receiver for a punctured CHPC with two optical correlators.

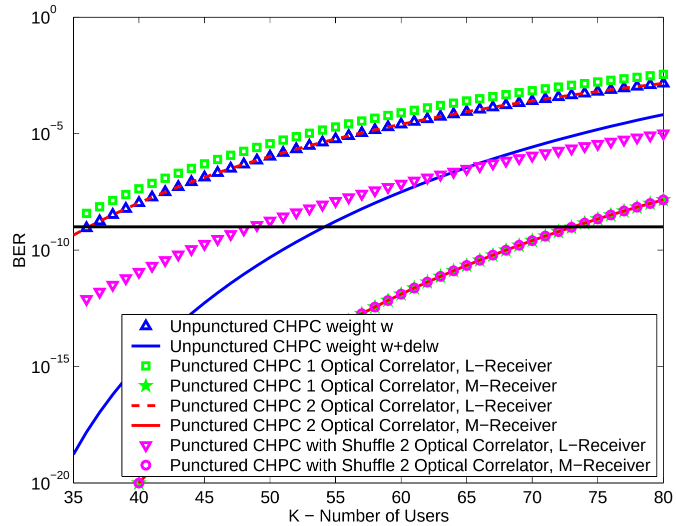

Fig. 4. Relationship between number of simultaneous users K, and BER for various punctured and unpunctured CHPC schemes with $w = 20$, $\Delta w = 12$, $T = 71$, $u_M = 1$ and $u_L = 6$.

$w+\Delta w$ available wavelengths with equal probability, we have the potential for significant performance improvements at the cost of increased complexity.

Puncture with One Optical Correlator For the most basic puncturing scheme, the receiver consists of a single optical correlator that passes $w + \Delta w$ wavelengths and two electrical thresholders (see Fig. 2). The hit probability is

$$q = \frac{w}{2T} + \frac{u_M}{u_M + u_L}\frac{\Delta w}{2T} \quad (3)$$

In order to see an improvement in the BER of the M-bits, we must have two separate thresholders at the receiver. The first thresholder, which we refer to as the L-thresholder, sees both the L-bits and the M-bits with BER given by (2) with $Th = w$ and q as in (3), while the M-thresholder can sees only the M-bits with BER given by (2) with $Th = w + \Delta w$ and q as in (3).

Puncture with Two Optical Correlators Here two separate receivers are used, a L-receiver with correlator passing w wavelengths and a thresholder set with $Th = w$, and an M-receiver with a correlator passing $w + \Delta w$ wavelengths and a thresholder set with $Th = w + \Delta w$, as shown in Fig. 3. The introduction of the second correlator improves BER performance as long as signal power is sufficiently high (so that even with the 3 dB power loss incurred when the signal is split between the two correlators, the system still operates so that performance is limited by multiple access interference (MAI) rather than by noise). The structure of the M-receiver is equivalent to that described in the previous scheme, and as such the BER is given by (2) with $Th = w + \Delta w$ and q as in (3). However, the introduction of the second correlator reduces the hit probabilities for the L-receiver to $q = \frac{w}{2T}$ and therefore improves the BER for the L-bits (given by (2) with $Th = w$) at the cost of increased receiver complexity and signal power.

Puncture with λ-shuffle and Two Optical Correlators For a λ-shuffle scheme, $w + \Delta w$ switches puncture any subset of Δw wavelengthes at the transmitter. The structure of a receiver with two optical correlators is essentially unchanged from that in Fig. 3, apart from the fact that for a reconfigurable system, user i's L-correlator must be tunable to the particular set of w unpunctured wavelengths used by user i for a particular message. (This may be achieved by inserting of $w + \Delta w$ switches which block wavelengths on a per-message (not per-bit) basis immediately after each delay line in the L-correlator (see Figs. 1 and 3), so that only the desired subset of w wavelengths is used for correlation.) The M-receiver has the same structure, and therefore the M-bits are recieved with the same BER as in the single optical correlator scheme. However, when we assume equiprobable puncturing of each set of Δw wavelengths, the hit probability for the L-receiver is reduced to $q = \frac{w}{2T}\left(\frac{u_M}{u_M+u_L} + \frac{u_L}{u_M+u_L}\frac{w}{w+\Delta w}\right)$, so that BER obtained from (2) with $Th = w$ improves significantly as well, again at the cost of increased complexity and signal power.

B. Comparison of schemes

Figure 4 shows the BER vs the number of simultaneous users for the schemes described above. As expected, for every puncturing scheme, the BER of the L-bits always exceeds that of the M-bits. Because puncturing reduces interference on the fiber channel, we see improved BER for the M-bits in a puncturing scheme as compared to an unpunctured scheme where all bits are transmitted with a weight $w+\Delta w$ CHPC. It follows that for a given maximum BER, puncturing the weight $w + \Delta w$ CHPC increases the number of users that can be accommodated by the system. For a puncturing scheme using one optical correlator, there is a performance penalty for the L-bits when compared to an scheme where all bits are sent with an unpunctured CHPC of weight w. The performance penalty is caused by the additional Δw wavelengths transmitted by the interferers's M-bits passing through the L-correlator. When the extra Δw wavelengths due to the M-bits are blocked by using a second optical correlator, we see the the performance of the system for the L-bits is equivalent to an unpunctured CHPC of weight w, and improves even further when λ-shuffle puncturing is used.

Fig. 4 illustrates how our schemes increase capacity. For the parameters in Fig. 4 ($w = 20$, $\Delta w = 12$, $T = 71$, $u_M = 1$ and $u_L = 6$), if we require that M-bits are transmitted with BER $\leq 10^{-9}$, the basic puncturing scheme with one correlator can accommodate up to 71 simultaneous users, with L-bits received with BER $< 5 \times 10^{-3}$. With λ-shuffle puncturing, the BER of of the L-bits improves to $< 5 \times 10^{-6}$. We can see the effect of puncturing when we note that when we require that all data bits are received with BER $< 10^{-9}$, the system can accommodate only 54 users with a weight $w + \Delta w$ CHPC and 33 users with a weight w CHPC.

IV. Conclusion

We have shown that punctured CHPCs can be used to match the spreading code used in a wavelength-hopping time-spreading OCDMA network to source data consisting of bits with unequal priority levels. When puncturing is used to reduce the weight of the codewords used to send less important data bits, the total interference on the fiber media decreases, so that the capacity of the system increases and the more important bits are recieved with better BER than the less important bits.

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